Horizontal Mergers and Partial Privatization in a Mixed Oligopoly I

Kenji Fujiwara*

This paper studies the effects of privatization of a public firm on the profitability and welfare in the presence of horizontal mergers by private firms. We show that (i) even a merger with a sufficiently large share can be unprofitable if the degree of privatization is small enough, that (ii) a merger increases outsiders' profit and that (iii) a merger is welfare-deteriorating.

Kenji Fujiwara

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1 Introduction

This paper studies how partial privatization affects a merger activity by private firms. For this purpose, we build a model of a mixed oligopoly comprising one public firm and an arbitrary number of private firms.

Most of the theoretical literature on horizontal mergers views Salant *et al.* (1983) as a point of departure. They show that a merger with less than 80% share is unprofitable. While their result is appealing from both a theoretical and a practical point of view, it rests on a number of simplifying assumptions including (i) linear demand, (ii) constant marginal cost, (iii) Cournot-Nash conjectures, (iv) a homogeneous product, (v) no

* School of Economics, Kwansei Gakuin University. Uegahara 1-1-155, Nishinomiya, Hyogo, 662-8501, Japan. Tel: +81-798-54-7066. Fax: +81-798-51-0944. E-mail: kenjifujiwara@kwansei.ac.jp.

cost synergy, (vi) a static setting, and (vii) absence of free entry.[1] It is well recognized that the result of Salant *et al.* (1983) can easily be modified if one or more of these assumptions are replaced.

This paper is also along this line of research, but our purpose is clearly different from that of the predecessors. While most of the previous works are interested in how the conclusion of Salant *et al.* (1983) is sensitive to the above assumptions, we examine how it is affected by the presence of privatization with all the other assumptions of Salant *et al.* (1983) kept intact. To our knowledge, there is little literature examining the effects of privatization on mergers in a context of a mixed oligopoly. Barcena-Ruiz and Garzon (2003) are the first to address this issue. Assuming a differentiated mixed duopoly based on an approach of Matsumura (1998) and Matsumura and Kanda (2005), they show that two parameters, one of which measures product differentiation and the other of which measures the degree of privatization, play an important role for the decision to merge. On the other hand, assuming away product differentiation, Coloma (2006) compares welfare levels in the cases with and without merger. Mendez-Naya (2008) proves a possibility of gains from merger in a mixed oligopoly model with an arbitrary number of private firms.[2]

While these papers substantially contribute to the literature, there is a room for a further analysis. This paper attempts to fulfill this gap in part. First, we allow for an arbitrary number of insider and outsider firms whereas Barcena-Ruiz and Coloma (2003) focus on a duopoly and Mendez-Naya (2008) presumes that only one private firm merges with the public

1) See Footnote 4 in Qiu and Zhou (2005, p. 39) for the literature relaxing (ii)-(v). Moreover, Fauli-Oller (1997) relaxes (i), Dockner and Gaunersdorfer (2001) and Benchekroun (2003) (vi), and Davidson and Mukherjee (2007) (vii) to revisit Salant *et al.*'s (1983) proposition.

2) Note that Barcena-Ruiz and Garzon (2003) assume only one private firm.

firm, i.e., the number of insiders is fixed to two. Second, we assume constant marginal cost where these authors assume a quadratic cost, namely, increasing marginal cost.[3] This simplification not only allows us to avoid numerical simulation but also makes our analysis comparable to Salant *et al.* (1983). Indeed, our analysis straightforwardly reduces to Salant *et al.*'s (1983) once one assumes away privatization in our model. Third, this paper is interested in the merger comprising only private firms, which also differentiates us from the existing literature focusing on the merger which involves the public firm.

The paper is structured as follows. Building a basic model, Section 2 briefly describes a pre-merger equilibrium and Section 3 a post-merger equilibrium. Then, Section 4 investigates how partial privatization influences merger profitability. Section 5 further examines the effect of a merger on outsiders' profit and welfare in the presence of a public firm. Section 6 concludes the paper.

2 A Pre-Merger Equilibrium

As a preliminary, this section briefly sketches the equilibrium before a merger. In order to make our argument as parallel to Salant *et al.* (1983) as possible, let us consider a canonical model of oligopoly with linear demand and constant marginal cost. The inverse demand function is linear and given by $p = a - X,\ a > 0$ where p and X are the price and industry output, respectively. There are potentially three types of firms: (i) one public firm (indexed 0), (ii) $m \geq 1$ firms participating in the merger (represented by i), and (iii) $n \geq 1$ outside firms (represented by j). All of these firms share the identical cost given by $cx,\ a > c \geq 0$, where c is a marginal cost and x is an output.

3) Most of the literature on mixed oligopoly commonly assumes increasing marginal cost.

While $m+n$ private firms maximize profits, the public firm maximizes the weighted sum of profit π_0 and welfare U by following the formulation of Matsumura (1998) and Matsumura and Kanda (2005):[4)]

$$\begin{aligned}
&\theta\pi_0 + (1-\theta)U \\
&= \theta\pi_0 + (1-\theta)\left(\frac{X^2}{2} + \pi_0 + \sum \pi_i + \sum \pi_j\right) \\
&= \pi_0 + (1-\theta)\left(\frac{X^2}{2} + \sum \pi_i + \sum \pi_j\right), \quad \theta \in [0,1], \qquad (1)
\end{aligned}$$

where $X^2/2$ gives consumer surplus and π_i and π_j are the profit of private firms. The profit of each firm is defined by

$$\pi_0 = (a-c-X)x_0 \qquad (2)$$

$$\pi_i = (a-c-X)x_i \qquad (3)$$

$$\pi_j = (a-c-X)x_j. \qquad (4)$$

The public firm chooses x_0 to maximize (1) whereas each private firm maximizes (3) and (4) in a Cournot-Nash fashion. Then, straightforward manipulations yield the equilibrium outputs and profits as follows.

$$x_0 = \frac{a-c}{\theta(m+n+1)+1}, \quad x_i = x_j = \frac{\theta(a-c)}{\theta(m+n+1)+1} \qquad (5)$$

$$\pi_0^{pre} = \theta\left[\frac{a-c}{\theta(m+n+1)+1}\right]^2 \qquad (6)$$

$$\pi_i^{pre} = \pi_j^{pre} = \left[\frac{\theta(a-c)}{\theta(m+n+1)+1}\right]^2, \qquad (7)$$

where the superscript pre indicates the pre-merger equilibrium.

3 A Merger Involving No Public Firm

In this section, we turn to the equilibrium after m private firms merge. After they merge, each insider firm maximizes the joint profit defined by $\sum \pi_i$. The equilibrium in this situation is characterized as

4) Strictly speaking, Matsumura (1998) adopts a slightly more general assumption that the public firm's objective function is a mere weighted sum of profit and welfare.

$$x_0 = \frac{a-c}{\theta(n+2)+1}, \quad x_i = \frac{\theta(a-c)}{m[\theta(n+2)+1]}, \quad x_j = \frac{\theta(a-c)}{\theta(n+2)+1} \tag{8}$$

$$\pi_0^{post} = \theta\left[\frac{a-c}{\theta(n+2)+1}\right]^2 \tag{9}$$

$$\pi_i^{post} = \frac{1}{m}\left[\frac{\theta(a-c)}{\theta(n+2)+1}\right]^2 \tag{10}$$

$$\pi_j^{post} = \left[\frac{\theta(a-c)}{\theta(n+2)+1}\right]^2, \tag{11}$$

where the superscript *post* stands for the post-merger equilibrium.

4 Partial Privatization and Merger Profitability

Having derived the equilibrium outputs and profits in the pre- and post-merger equilibria, this section examines how merger profitability is affected by the presence of partial privatization. The merger profitability is defined by the difference between the post- and pre-merger profits: $m\pi_i^{post} - m\pi_i^{pre}$. Given (7) and (10), the profitability is obtained as

$$\begin{aligned} & m\pi_i^{post} - m\pi_i^{pre} \\ &= \left[\frac{\theta(a-c)}{\theta(n+2)+1}\right]^2 - m\left[\frac{\theta(a-c)}{\theta(m+n+1)+1}\right]^2 \\ &= \frac{\theta^2(a-c)^2\Delta}{[\theta(n+2)+1]^2[\theta(m+n+1)+1]^2}, \end{aligned} \tag{12}$$

$$\Delta \equiv [\theta(m+n+1)+1]^2 - m[\theta(n+2)+1]^2. \tag{13}$$

The rest of this section examines the properties of Δ. In order to confine attention to the role of privatization θ, let us rewrite (13) as follows.

$$\Delta = \theta^2[(m+n+1)^2 - m(n+2)^2] - 2\theta(m-1)(n+1) - (m-1). \tag{14}$$

Noting that the second and third terms are both negative in the right-hand side of (14), we have the two subcases: (i) $(m+n+1)^2 - m(n+2)^2 > 0$ and (ii) $(m+n+1)^2 - m(n+2)^2 \leq 0$. It is trivial that $\Delta < 0$, i.e., a merger is never profitable in Case (ii). This trivial result is stated in:

Proposition 1. *Suppose* $(m+n+1)^2 - m(n+2)^2 \leq 0$ *or equivalently* $1 \leq m \leq (n+1)^2$. *Then, a merger is unprofitable for any* $\theta \in [0,1]$.

While someone may find little interest in this proposition, it is useful to comment this case briefly. As Salant *et al.* (1983) show, a merger with a small share is unprofitable. The same is true of the present setting with a public firm. In this sense, the degree of privatization plays no active role for unwillingness to merge for firms with small market shares. However, what should be stressed is that such unwillingness is stronger in the presence of a public firm. This is because a public firm has a larger share than private firms. Accordingly, the insider firms become more unwilling to merge in the presence of a public firm than in the absence of it.

Let us move on to the case where $(m+n+1)^2 - m(n+2)^2 > 0$ or $m > (n+1)^2$.[5] Then, the degree of privatization proves to play an important role, which is summarized by:

Proposition 2. *Suppose* $m > (n+1)^2$. *Then, a merger is profitable if and only if*

$$\theta > \widetilde{\theta} \equiv \frac{(m-1)\left(n+1+\sqrt{m}\right)}{(m+n+1)^2 - m(n+2)^2}. \tag{15}$$

Proof. In view of (14), we see that $\Delta > 0$ if and only if

$$\begin{aligned}\theta &> \frac{(m-1)(n+1) + \sqrt{(m-1)^2(n+1)^2 + (m-1)\left[(m+n+1)^2 - m(n+2)^2\right]}}{(m+n+1)^2 - m(n+2)^2} \\ &= \frac{(m-1)\left(n+1+\sqrt{m}\right)}{(m+n+1)^2 - m(n+2)^2},\end{aligned}$$

which is the threshold for θ in (15). **Q.E.D.**

5) Mathematically speaking, $(m+n+1)^2 - m(n+2)^2 > 0$ is equivalent to $m < 1$ and $m > (n+1)^2$. However, $m < 1$ means that no firm merges, which is meaningless and thus we can ignore it.

As mentioned in Introduction, Salant *et al.* (1983) show that a merger is profitable if its share is sufficiently large. However, we need not only a sufficiently large share of the merged firm but also a sufficiently large degree of privatization for a positive profitability of merger. The intuition behind this finding is as follows. When θ is small enough, the public firm's share becomes large relative to private firms.[6] Therefore, even a large-shared merger can be unprofitable. On the contrary, if the public firm is sufficiently privatized (θ is large enough), the story of Salant *et al.* (1983) revives; a merger can be profitable if its share is sufficiently merge.

It is fair to say that θ is fairly low in developing countries as compared to developed countries. In these countries, a merger can be unprofitable even if its market share is sufficiently large. In this sense, we can claim that it is more difficult for a merger to emerge in developing countries than in developed countries. Proposition 2 is insightful since it states that the presence of insufficiently privatized public firm serves as a tacit anti-trust policy.

Remark 1. In the above proposition, we have allowed for an arbitrary number of insider and outsider firms. Then, someone may naturally wonder what will happen if the number of insiders is equal to that of outsiders. We briefly address this question. Under $n = m$, Δ in (13) simplifies to

$$\Delta|_{n=m} = -(m^3 - 1)\theta^2 - 2(m + 1)(m - 1)\theta - (m - 1) < 0,$$

that is, a merger necessarily becomes unprofitable. In other words, the degree of privatization is irrelevant for unwillingness to merge.

6) In the extreme case with $\theta = 0$ (welfare-maximizing public firm), the public firm naturally charges its price equal to marginal cost and hence all the private firms exit the market.

Remark 2. Let us look at how $\widetilde{\theta}$ in (15) is affected by a small change in m and n. The effect of an increase in m on $\widetilde{\theta}$ is ambiguous, but an increase in n definitely raises $\widetilde{\theta}$, i.e., profitability of a merger becomes more unlikely. This is easily interpreted by invoking Salant *et al.* (1983).

5 Effects of Mergers

The foregoing argument has focused on the profitability of insider firms. This section turns to the other effects of a merger. We begin by considering the effect on the outsiders' profit and then proceed to the welfare effect.

5.1 Effects on Outsiders' Profit

From (7) and (11), the ratio between the post- and pre-merger profit of an outside firm is

$$\frac{\pi_j^{post}}{\pi_j^{pre}} = \left[\frac{\theta(m+n+1)+1}{\theta(n+2)+1}\right]^2,$$

which enables us to prove:

Proposition 3. *Any merger is profitable for outsiders.*

Proof. $\pi_j^{post}/\pi_j^{pre} > 1$ is equivalent to $m > 1$, which is always satisfied by assumption. Hence, the proposition follows. **Q.E.D.**

This is familiar in the literature. As Salant *et al.* (1983) and their successors clarify, a merger inevitably induces a free rider incentive for outsider firms. In other words, any outsider gains from the merger by expanding its own share. This is because (i) the merger raises price and (ii) outsiders produce more than before the merger by reacting to the merger formation. The same survives the present case with a public firm.

5.2 Welfare Effects

Under the assumptions and specifications in consideration, welfare is given by

$$U = \frac{X^2}{2} + \pi_0 + m\pi_i + n\pi_j, \tag{16}$$

where the first term is consumer surplus. Substituting (5)-(7) into (16) yields the pre-merger welfare:

$$U^{pre} = \frac{[\theta(m+n)+1][\theta(m+n+2)+1](a-c)^2}{2[\theta(m+n+1)+1]^2}. \tag{17}$$

On the other hand, substituting (8)-(11) into (16), we have the post-merger welfare:

$$U^{post} = \frac{[\theta(n+1)+1][\theta(n+2)+1](a-c)^2}{2[\theta(n+2)+1]^2}. \tag{18}$$

Eqs. (17) and (18) help us to find the welfare effect of a merger:

Proposition 4. *A merger among private firms always reduces welfare for any* $\theta \in (0,1]$.

Proof. Taking the ratio of U^{post} and U^{pre}, we have

$$\frac{U^{post}}{U^{pre}} = \frac{[\theta(m+n+1)+1]^2[\theta(n+1)+1][\theta(n+2)+1]}{[\theta(n+2)+1]^2[\theta(m+n)+1][\theta(m+n+2)+1]}.$$

Subtracting the denominator from the numerator yields

$$\begin{aligned}
&(\text{numerator}) - (\text{denominator}) \\
&= \theta\left\{-\theta^3(n+2)[(m+n+1)(m+n-1)+2m+n]\right. \\
&\quad \left.-\theta^2[(m+n+1)(m+3n+3)+2(m-1)] - \theta(2m+2n+3) - 1\right\}.
\end{aligned}$$

Therefore, it follow that $U^{post} = U^{pre}$ in the limiting case with $\theta = 0$ and that $U^{post} < U^{pre}$ for any $\theta \in (0,1]$. **Q.E.D.**

Under $\theta = 0$, the market is monopolized by the nationalized firm which charges price equal to marginal cost. Hence, there is neither merger nor

net change in welfare. In contrast, welfare necessarily declines due to a merger since its negative effect coming from monopolization dominates the gains for outsiders. This situation in a context of private oligopoly is also illustrated in Figure 4 of Salant *et al.* (1983, p. 196).

6 Concluding Remarks

While most of the existing literature on horizontal merger is concerned with how relaxing one or more of the simplifying assumptions made in Salant *et al.* (1983) modifies their striking result, we pay attention to the role of partial privatization. We have identified that not only the share of the merged firm but also the degree of privatization is an important factor for the decision to merge.

In this paper, we have strategically made all of the assumptions in Salant *et al.* (1983) intact so as to make our result comparable to theirs and to avoid numerical simulations which Barcena-Ruiz and Garzon (2003) and Mendez-Naya (2008) make. However, we must recognize that our result will be inevitably criticized since it rests on a number of restrictions as in Salant *et al.* (1983). It is our future research agenda to consider the robustness of our result in a more general setting. In the sequel to this paper, we will consider another possibility in which the merger involves the public firm.

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